A Transition in the Dynamics of a Diffusive Running Sandpile

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Abstract

The dynamics of a running sandpile is shown to undergo a dynamical transition as diffusion is increased from zero. The transition takes place after the local diffusion has become so large as to erase the local inhomogeneities, caused by the intermittent rain of sand, before they can trigger avalanche activity. The system then undergoes an abrupt change with the self-similar structure of the dynamics being replaced with quasi-periodic, near system-size transport events. These results may have significant implications for many of the driven physical systems for which SOC-based dynamical models have been proposed.

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A competition between transport mechanisms with very different characteristic scale lenghts are common in both neutral fluids and plasmas. A typical example is the transport of particles through a set of vortices. Such a situation leads to an intermediate asymptotic regime with transport properties, which combine disparate effects of diffusive and ballistic transport.

Similar situations may be encountered in systems where the dominant transport mechanism in operation is through avalanches, as in systems whose dynamics are governed by self-organized criticality (SOC) [1]. As an example, this is the case when collisional diffusion is also present in the system. Hybrid regimes might then be expected in which, at the same time that the transport changes, the intrinsic nature of the system dynamics may also be altered. One system in which one might expect these hybrid regimes is a magnetically confined plasma. There is significant experimental evidence in these plasmas supporting the self-similar nature of plasma fluctuations and the existence of avalanche-driven transport [2,3]. Perhaps more importantly, the SOC model proposed for the dynamics of plasma transport [4,5] has provided a framework in which some previously unexplained experimental facts [6–9] might be understood. But there are fundamental reasons to expect that diffusion of particles and energy down the system gradients should also play an important role in the dynamics. Furthermore, this is not a special case. The large number of models based on the idea of self-organized-criticality that have been proposed in the last decade [10–13], together with the near universal character of the diffusion processes, suggests that the impact that diffusion (or other transport mechanisms) may have on SOC systems should be of very broad interest.

In this paper we argue that neglecting diffusion can indeed lead to a reduction of the range of dynamics found in the SOC-like models and to a narrowing of their relevance for the description of physical systems. In particular, we show how it can strongly modify the SOC dynamics of a SOC system even while remaining a very subdominant transport mechanism. A dynamical transition takes place in the system as the relative importance of diffusive transport increases beyond a critical threshold. The role of the avalanche-like transport events characteristic of the self-similar SOC state is then abruptly taken over by quasi-periodic constant-size edge-triggered events. As a result, the system loses its "self-organized critical" properties. In retrospect, this conclusion might perhaps be foreseen. Thinking of diffusion as an external drive for the system, it could, if sufficiently strong, plausibly drive the system out of the absorbing state characteristic of SOC dynamics [14]. We will, however, show that the required diffusional component needed for this to happen is surprisingly small.

To study the interaction of diffusive and avalanche transport, we have used a driven directed running sandpile [15,16] with an added diffusive flux. The sandpile is composed of L cells, each labeled by an integer index, j. Each cell stores some amount of sand, h_j, which is a continuous variable. N_f grains of sand are moved to the next cell, h_{j+1}, whenever the local slope, Z_j, exceeds a prescribed critical value, Z_c. An open boundary exists at $j = L$, and a closed boundary at $j = 1$. The random drive is provided by dropping a grain of sand at each iteration with probability P_0. Finally, the net amount of sand diffusively leaving (or entering, if positive) the j^{th} cell is $\Gamma_j = D_0(Z_j - Z_{j-1})$. D_0 is the diffusion coefficient. In steady state, the sandpile domain is divided into two regions: 1) an inner or upper ("diffusive") region, where the transport is carried only by the diffusive flux and where the slope, which is linear with j, stays below $Z_c - N_f$; and 2) an outer or lower ("SOC-like") region, where transport is still dominantly driven by avalanches. These regions meet at cell j_t, which can be estimated from a flux balance calculation. This is because diffusive transport is the only one possible for $j < j_t$, a steady state could only be reached if all falling sand collected for cells $1 \leq j \leq j_t$ can be diffusively injected into the avalanche dominated region, i.e., if $P_0 \, j_t \simeq D_0(Z_c - N_f)$. Note that the directed running sandpile is one in which the steepest gradient direction is deterministically used to define the overturning direction and the driving rate allows the hydrodynamics regime to be accessed [16].

In this model we find a dynamical transition that takes place when the fraction of transport diffusively driven out of the sandpile is still very much below unity (as low as $10^{-3}-10^{-4}$ at the transition point, as estimated using $D_0 Z_C / P_0 L$ [17]). Diffusion is thus an extremely subdominant transport mechanism. The spatial region where all the interesting dynamics take place is the SOC-like region. There, the dynamical regime prior to the transition is marked by all the classic SOC characteristics. The averaged slope is constant and can be estimated as $\overline{Z} \sim Z_c - N_f/2$ [18] (see solid line in Fig. 1). There is a nearly uniform distribution of avalanche initiation positions and ending positions (see upper line in Fig. 2, where the probability of an avalanche starting at cell j is plotted against j). Additionally, clear power-law tails are exhibited by the size probability distribution function (PDF) and the frequency spectra, revealing self-similar structures over many scales of size and time. After the transition, the new dynamical regime is markedly different: the slope becomes linear with j, staying in the range $Z_c - N_f < \overline{Z} < Z_c - N_f/2$ (see dashed line in Fig. 1). The PDFs of avalanche initiation and termination locations become very narrow (lower line in Fig. 2). Finally, the self-similarity is lost.

The controlling parameter for the transition seems to be $\kappa = D_0 N_f^2/P_0$, a combination of the drive, diffusion, and overturning size. Its physical meaning is intimately related to the sandpile slope roughness. This is estimated differently in the first and second dynamical regimes:

$$R^2 = \overline{\left\langle (Z - \langle Z \rangle)^2 \right\rangle} \qquad \text{and} \qquad R^2 = \overline{\left\langle \left(\frac{dZ}{dx} - \left\langle \frac{dZ}{dx} \right\rangle \right)^2 \right\rangle} \tag{1}$$

where the overbar denotes time-averaging and the brackets denote space-averaging over those cells in $j_t \leq j \leq L$. Notice that these definitions differ from the usual one (given by the variance of the height profile [19]) in that the linear or parabolic trends of the height profiles have been removed. As a result, there is no scaling of the roughness with the system size. This roughness can be estimated, in the absence of diffusion, by $R^2 \simeq N_f^2/12$ [18]. In the strongly-diffusive limit, the falling sand lumps will be eroded away completely before they can trigger any avalanche activity. Therefore, it can be estimated as $R^2 \propto P_0/D_0$ by assuming a fixed underlying slope state and a characteristic time for diffusion of the lump over the whole sandpile given by $T_d \sim L^2/4D_0$. κ thus represents the quotient of both limits, and it is an indirect measure of which of them dominates the dynamics. In Fig. 3,

R/N_f is plotted against $\kappa = P_0/D_0 N_f^2$ for sandpiles with different system sizes (the drive has been set to $P_0 = 5 \cdot 10^{-4}$ in order to minimize avalanche overlapping, which would distort any a posteriori single-avalanche analysis). The collapse of all curves onto a single one, with the two limits clearly identified, confirms the previous estimations. A good fit to R/N_f can be found using $R/N_f = a_1/(a_2 + \kappa^2)^{1/4}$, where $a_1 = 0.725$ and $a_1 = 22$ (see Fig. 3). Therefore, specifying κ, the external control parameter, also fixes the roughness, which provides a clearer physical interpretation of the mechanisms behind the transition.

As we increase κ the dynamics of the avalanches sharply changes, which defines the transition point. From avalanches with a broad distribution of sizes starting nearly uniformly over the whole sandpile, to quasiperiodic avalanches of the maximum possible size starting from the edge. This change is illustrated in Fig. 2 where three representative examples of the PDF of the initiation points have been plotted. There are several measures that illustrate this transition in the dynamics. We have used the width of this PDF normalized to the SOC-region size, which is plotted against κ for several system sizes in Fig. 4. It exhibits a sharp jump around $\kappa \simeq 22$, which can be seen not to be an effect of the logarithmic scale. For low-κ values, the standard deviation is large and constant. The average initiation position (not shown) is close to the center of the avalanche-dominated region, since avalanches start and stop all across the region. The noticeable upward spike exhibited by the standard deviation prior to the transition is not an artifact. It is indicative of a sharp onset of a new contribution to the PDF: that associated to the large events triggered at the lower end of the pile (see middle line of Fig. 2). As κ increases across the critical value, these edge-triggered avalanches come to dominate. They propagate all the way up the sandpile through the SOC-like region, which causes the sharp drop of the normalized standard deviation seen in Fig. 4. We then conclude that the system enters the quasi-regular regime after the roughness has decreased below a critical value, given by the implicit equation $\kappa(R_c) \simeq 22$. The fact that the system size does not enter in the definition for κ, and no system size scaling has been found suggests that the transition may not be a phase transition. For this reason we have used the term dynamical transition.

The main differences between the two dynamical regimes can be seen most clearly in the dynamics of the non-diffusive transport. In Fig. 5, two time traces for the number of avalanching sites are shown. The left one displays the multi-amplitude multi-time scale signatures characteristic of SOC systems. The right trace exhibits transport events that look like quasi-periodic relaxation oscillations of relatively constant amplitude. Physically, the roughness has been reduced, by the increasing diffusion, to the point in which no stopping position for avalanches exists throughout the SOC-like region. Likewise, no starting position will exist either. Diffusion is now capable of smoothing out any drop before another one is likely to fall, preventing an increase in the local roughness. However, the lower end of the sandpile is closer to the critical slope than any other location. This is due to both the larger flux coming through locations near the bottom and an asymmetry introduced by the edge itself [17]. Therefore, avalanches will always be initiated at the lower end, propagating all the way up to the diffusive region. Effectively all of them will transport mass out of the system. This type of avalanche then helps to provide the positive feedback mechanism needed for a sharp transition. Since they do not start or stop in the SOC-like region they can not alter the surface roughness. This allows for an even more rapid smoothing (remember that the rain drops must always be smaller then the avalanche chunks, N_f, for the system to display SOC dynamics). In this way, the SOC-like region can develop a diffusive profile (Fig. 1 dashed line). But most of the transport out of the system is still carried by avalanches. This is caused by the limited amount of diffusive transport allowed by an edge whose slope must stay below Z_c. This mechanism is therefore insufficient to balance the system drive, allowing the sand buildup that causes the periodic bursts.

In conclusion, in this letter we have shown that the interplay between a continuous diffusive transport channel and the avalanche channel can cause a transition in the dynamics of a self-organized critical system. The intrinsic robustness of the SOC paradigm can therefore be compromised. This can happen even if the alternative mechanism is very subdominant, which would usually justify its neglect in the description of the system dynamics. In this sense, we think that these results suggest that some of the SOC paradigms proposed for various systems of the physical and earth sciences [4,5,11–13] should perhaps be revisited, since in many cases diffusion (or some other mechanism) is an unavoidable element of the dynamics.

As a final comment, we think that rather than casting doubt on the usefulness of the SOC paradigm, these extended models could in fact increase the usefulness of SOC-like models in describing and understanding physical systems by extending the type of dynamics captured within the model framework. For example, this type of extension might find application in the description of tectonic systems, where creep or plastic deformations may be modeled (to first order) as a diffusive-like response to stress buildup. In this way, previously unreachable dynamical regimes may be now accessed within the same model.

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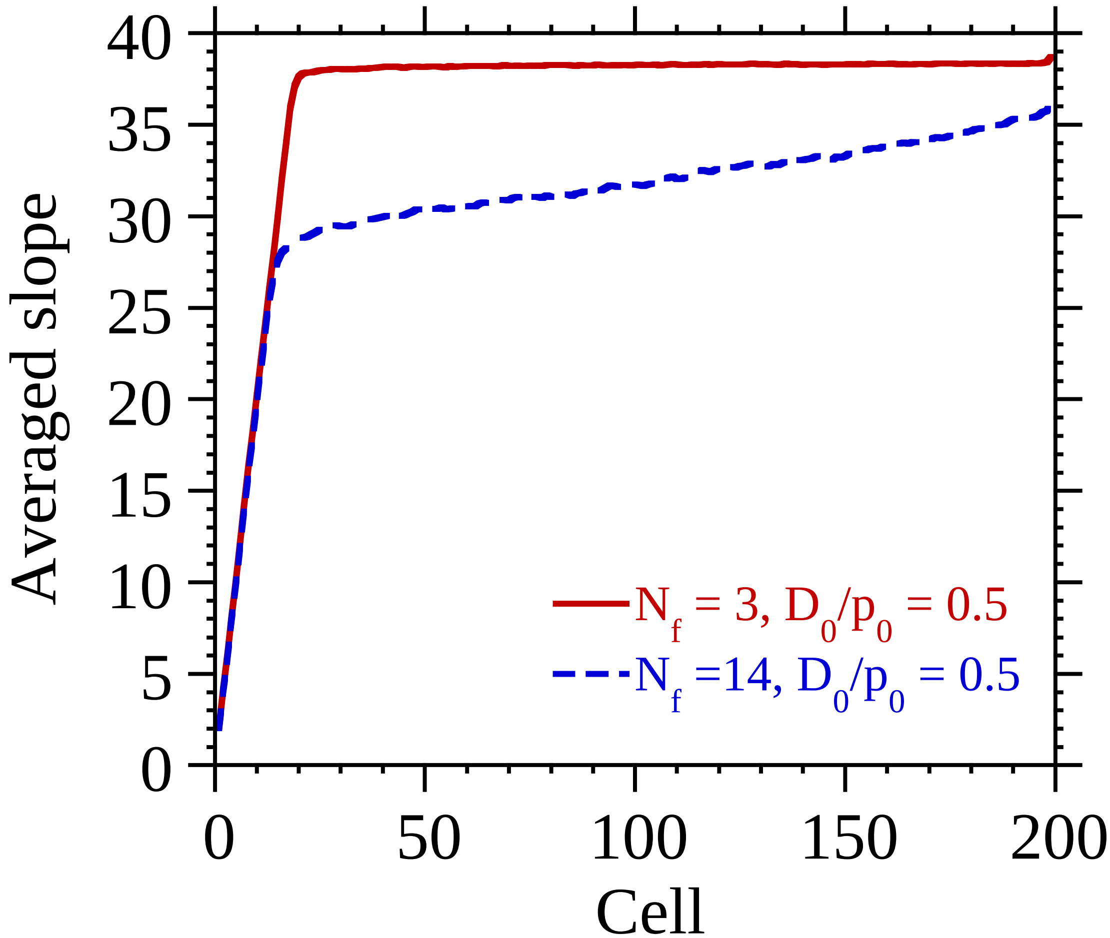

FIG. 1. Slope profiles before (solid line) and after (dashed line) the transition for a sandpile with $L = 200$.

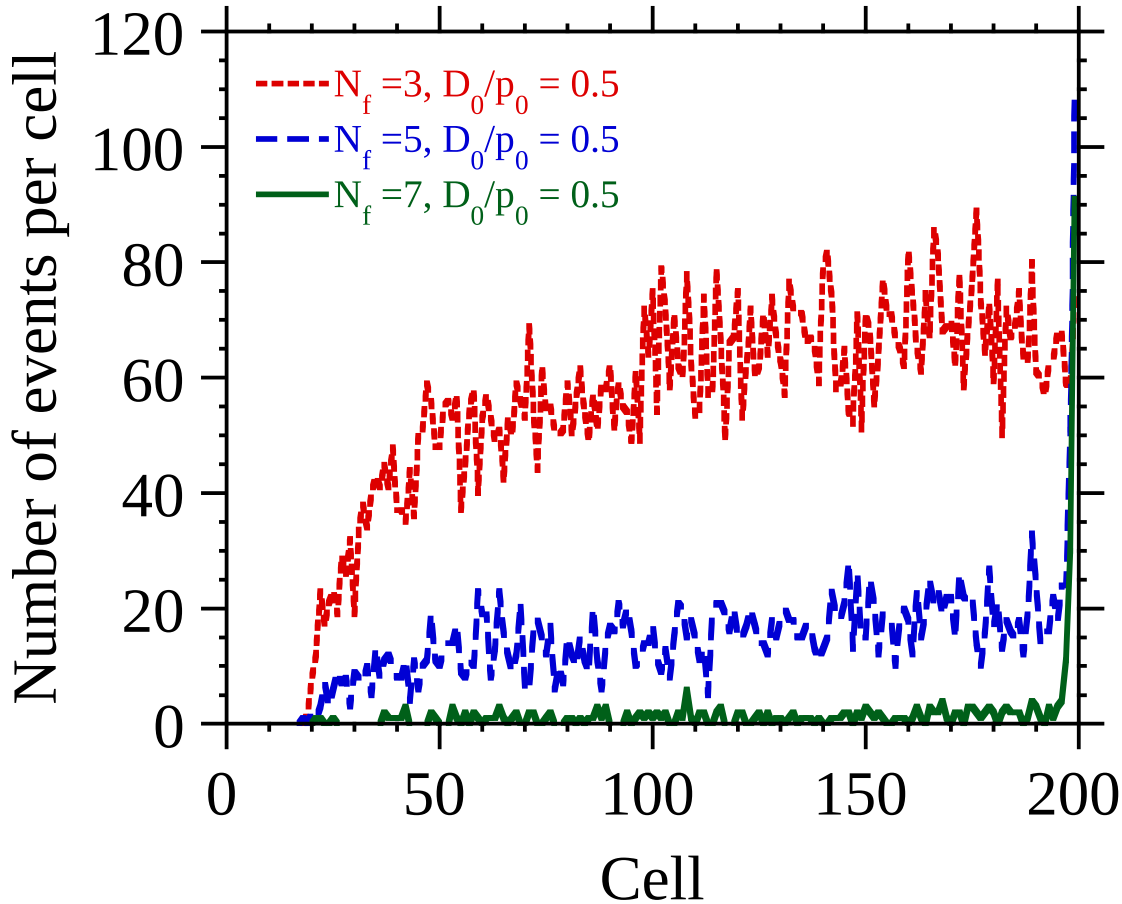

FIG. 2. Avalanche initiation point PDFs (see text for explanation) before (top line), near (middle line) and past (lower line) the transition for a sandpile with $L = 200$.

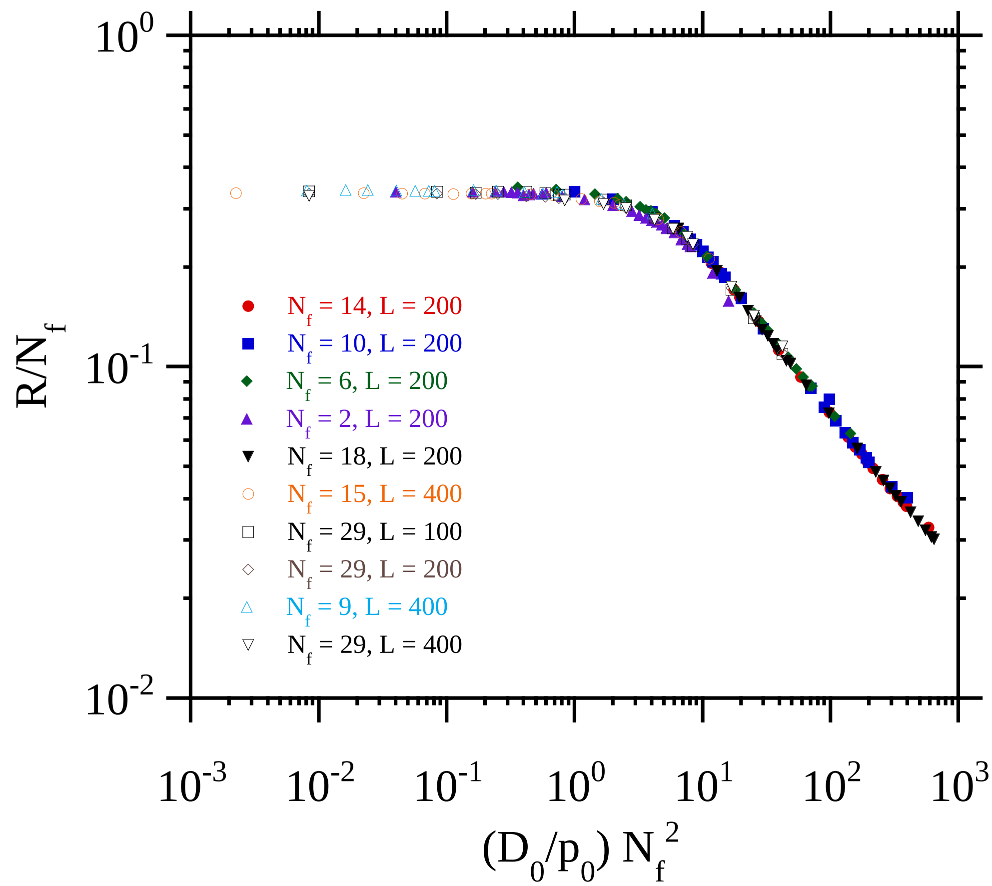

FIG. 3. Sandpile roughness (normalized to N_f) as function of κ for varying values of L and N_f.

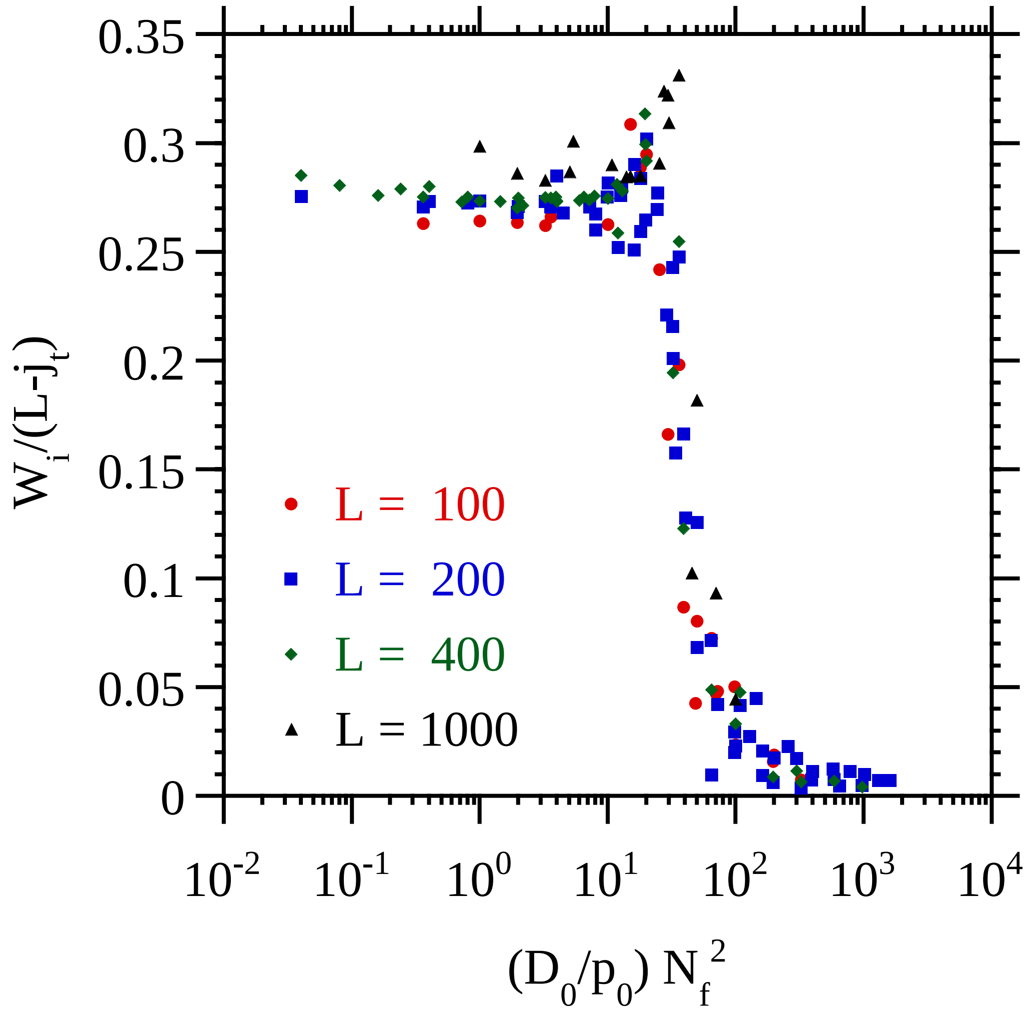

FIG. 4. Standard deviation of the initiation point PDF (normalized to the size of the SOC-region) as a function of κ for several values of L and N_f.

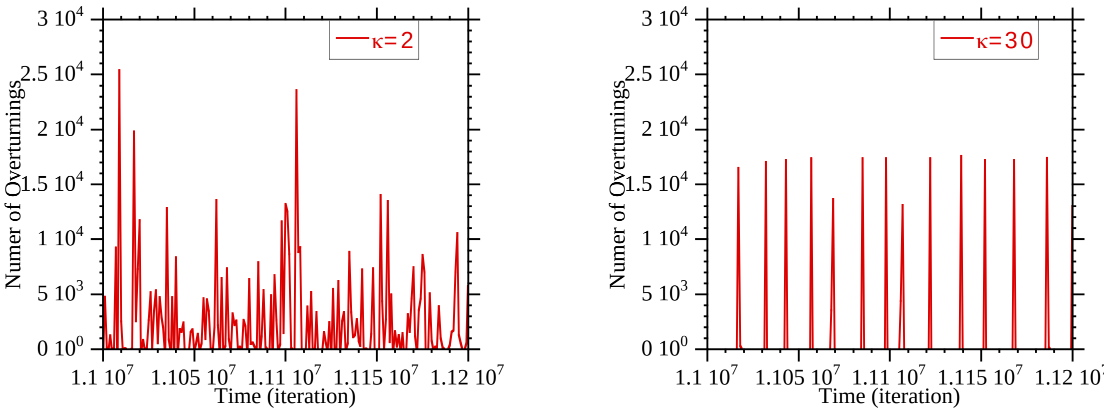

FIG. 5. Time trace of the number of avalanches sites before (left) and after (right) the transition for a sandpile with $L = 200$.