Flat Belt Friction

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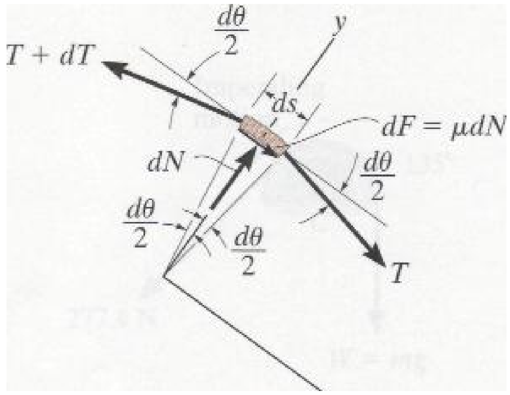
Let us examine the picture taken from Engineering Mechanics by R. C. Hibbeler:



Using this free body digram we will first sum the forces along both the Y and X axis's. Just as a note these are parallel and perpendicular to the normal force and we will be assuming that there is no acceleration (in other words the net forces are zero):

$$\sum F_y = 0 = T\cos(\frac{d\theta}{2}) + \mu dN - (T + dT)\cos(\frac{d\theta}{2})$$
$$\sum F_x = 0 = dN - (T + dT)\sin(\frac{d\theta}{2}) - T\sin(\frac{d\theta}{2})$$

Now since $d\theta$ is differentially small we can say that $\sin(\frac{d\theta}{2}) = \frac{d\theta}{2}$ and $\cos(\frac{d\theta}{2}) = 1$. So our equations simplify to:

$$\sum F_y = 0 = T + \mu dN - (T + dT) \qquad \Longrightarrow dT = \mu dN$$
$$\sum F_x = 0 = dN - (T + dT)\frac{d\theta}{2} - T\frac{d\theta}{2} \qquad \Longrightarrow dN = Td\theta - \frac{dTd\theta}{2}$$

But since the product of two differentially small numbers is negligible when compared to single differentially small number the second equation and be simplified to $dN = Td\theta$. Combining these equations (by eliminating dN) we obtain an equation which we will integrate to obtain our final result.

$$\frac{dT}{T} = \mu d\theta$$
$$\int_{T_1}^{T_2} \frac{dT}{T} = \mu \int_0^{\beta} d\theta$$
$$\ln\frac{T_2}{T_1} = \mu\beta$$

$$T_2 = T_1 e^{\mu\beta}$$

And this is the result which is displayed on the webpage.