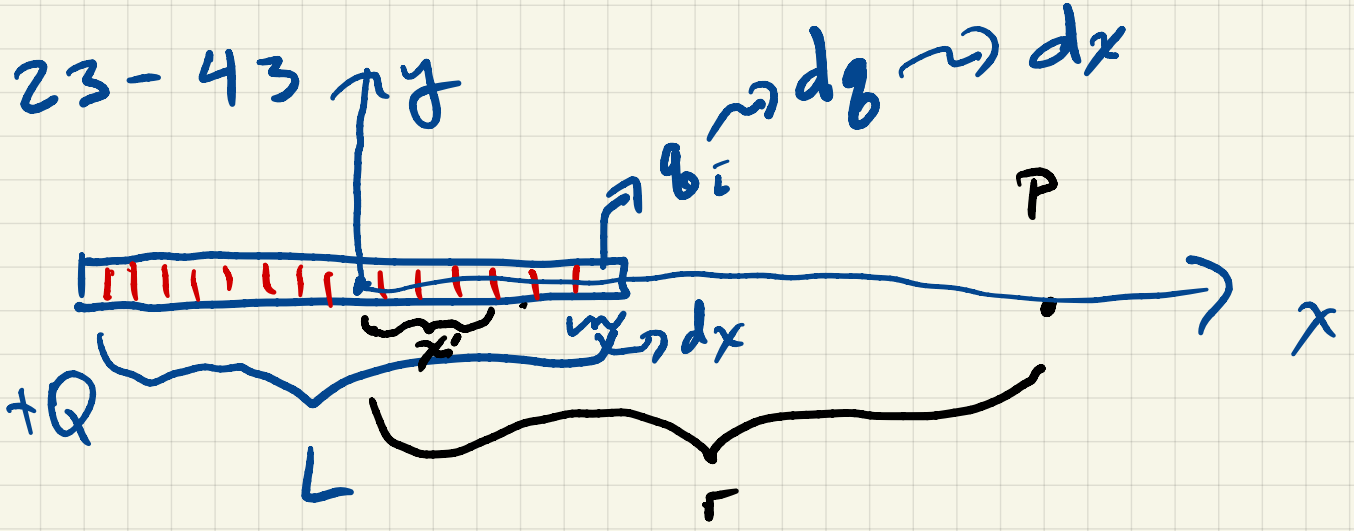
Homework help 2-24-21

23-43



Sub

$g = (r - x')$

$dg = -dx'$

$E_y = 0 \qquad E_x = E = \sum E_i$

$\lambda = Q/L \Rightarrow dq = \lambda\, dx = \lambda\, dx'$

$$dE_x = \frac{1}{4\pi\epsilon_0}\frac{dq}{(r-x')^2} = \frac{1}{4\pi\epsilon_0}\frac{\lambda\, dx'}{(r-x')^2}$$

$$E_x = E = \frac{1}{4\pi\epsilon_0}\int_{-L/2}^{L/2}\frac{dx'}{(r-x')^2} = \frac{\lambda}{4\pi\epsilon_0}\frac{1}{(r-x')}\Bigg|_{-L/2}^{L/2}$$

$$E_x = \frac{Q/L}{4\pi\epsilon_0}\left[\frac{1}{r-\frac{L}{2}} - \frac{1}{r+\frac{L}{2}}\right]$$

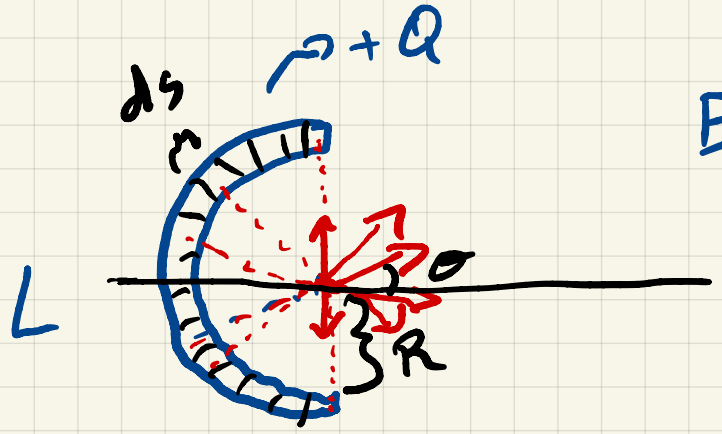
$$E_x = \frac{Q/L}{4\pi\epsilon_0}\,\frac{r+\frac{L}{2}}{r^2-\frac{L^2}{4}} - \frac{r-\frac{L}{2}}{r^2-\frac{L^2}{4}} = \frac{Q/L}{4\pi\epsilon_0}\,\frac{L}{r^2-\frac{L^2}{4}}$$

$$E = \frac{Q}{4\pi\epsilon_0\left(r^2-\frac{L^2}{4}\right)}$$

b)

If $r >> L$ $\quad r^2 >> \frac{L^2}{4}$ $\Rightarrow$ $E = \frac{Q}{4\pi\epsilon_0 r^2}$

23-47)



$E_y = 0$ by symmetry

$$E = E_x = \int dE \cos\theta \qquad dE = \frac{dq}{4\pi\epsilon_0 r^2}$$

$r = R$

$$dq = \lambda\, ds = \lambda R\, d\theta$$

$$ds = R\, d\theta$$

$$E = \int_{-\pi/2}^{\pi/2} \frac{\lambda R \cos\theta\, d\theta}{4\pi\epsilon_0 R^2} = \frac{\lambda R}{4\pi\epsilon_0 R^2} \int_{-\pi/2}^{\pi/2} \cos\theta\, d\theta$$

$$E = \frac{\lambda}{4\pi\epsilon_0 R} \sin\theta \Big|_{-\pi/2}^{\pi/2} \hat{x} = \frac{2\lambda}{4\pi\epsilon_0 R} \hat{x} = \frac{2Q}{4\pi\epsilon_0 L R} \hat{x}$$

$$L = \pi R \Rightarrow R = \frac{L}{\pi}$$

$$E = \frac{2\pi Q}{4\pi\epsilon_0 L^2} \hat{x}$$